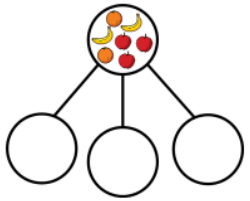
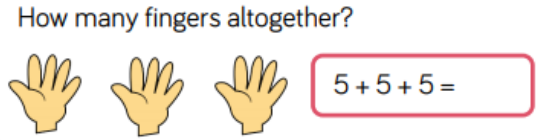
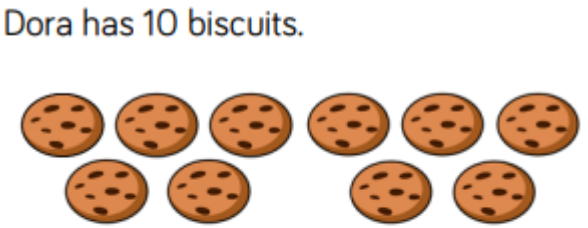
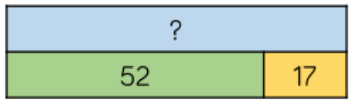
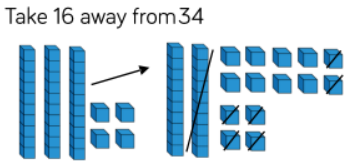
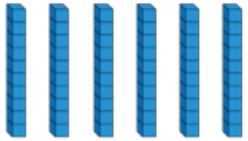
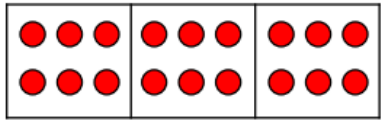
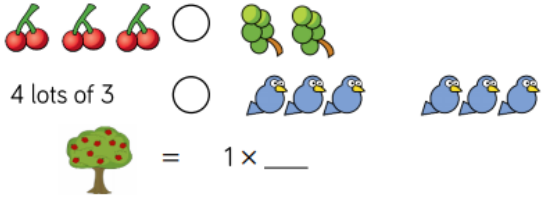
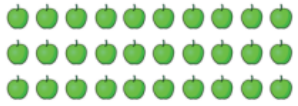
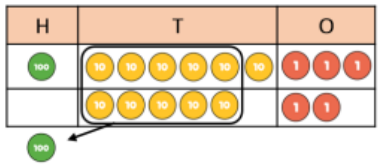
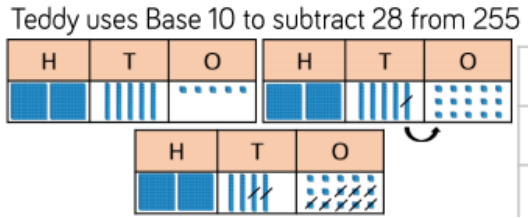
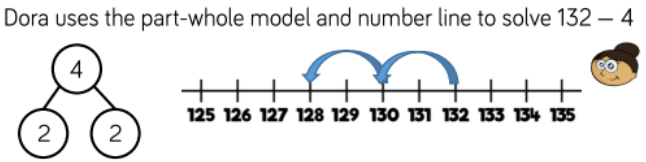
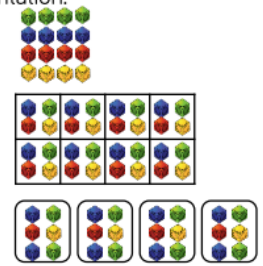
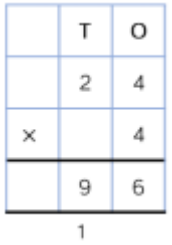
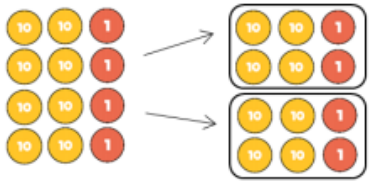
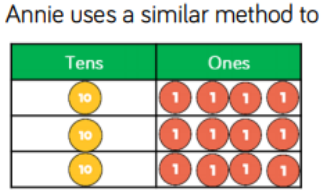
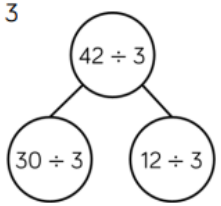
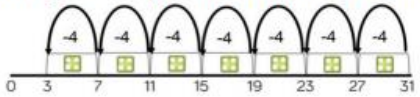
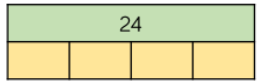
Year 1	Concrete	Pictorial	Abstract
Addition	Count out two sets of objects to match a verbal or written number sentence then count the total, or count ON from the first value/ group.  $5 = 3 + 2$ Select Numicon pieces to match a calculation e.g. for 4+2= select the 4 piece and the 2 piece. Count the total holes, or count ON from the value of the first piece. When working within 20, an initial amount will be made from two pieces of Numicon e.g. a 10 piece and a 3 piece to make 13, drawing on place value knowledge. Use objects to create a physical part-whole model, for example when working with number bonds to 10. Manipulate the objects to find different combinations. Place objects of two different colours into tens frames to investigate number bonds.	Draw a part-whole model for number bonds or to solve a word problem where two values are given to be combined. Use e.g. dots/ spots/ crosses to represent the values/ objects. Draw the separate items, count up and write or represent the total. Draw two sets of objects/ representations then count up the total e.g. for 6+2, draw 6 things then 2 things then count it all up (or count ON from the initial 6). Draw jumps on a number line to solve a calculation e.g. for 12+3, mark the 12, make three jumps of 1, and mark where you land – this is the total of the two values. Draw sticks and bricks (lines and dots) to represent Dienes. Partition using part-whole models. 	Counting on in your head or on your fingers (put the first number in your head and the second on your fingers) to find a total of two values – MENTAL addition. Use know addition facts to create fact families: 5+2=7 so 2+5 =7; 7-5=2; 7-2=5 Use known number bonds to solve missing number problems: 6 + ☐ = 8 Use number bonds to 10 to derive number bonds to 20: If 5+2=7, then 15+2=17 and 5+12=17 And use these to solve missing number problems. Draw part-whole models with numerals
Subtraction	Count out and take away objects to match a verbal or written number sentence then count how many are left e.g. for 9-2, first count out 9 objects then take away 3, then count the remaining objects – 6. Tens frames – put objects on to fill the frame then take the required amount off to match the subtraction. Use bead strings to move beads across from the starting amount, then counting how many now remain.	Draw objects on a whiteboard (or representations such as spots or crosses) then rub out or cross off the amount to be subtracted, counting the number remaining. Jump back on a number line to solve a subtraction. Use partitioning to cross a 10 when jumping back on a number line. Jump up on a number line to find the difference.	Counting back in your head or on your fingers to solve a subtraction. Use knowledge of fact families and inverse operations to derive subtractions from know additions. Solve missing number problems using known facts, counting back or counting up: 9 - ☐ = 4

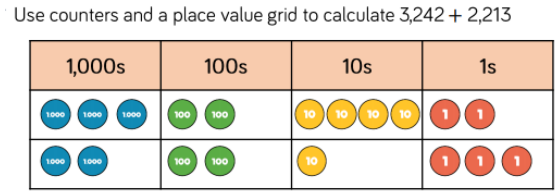
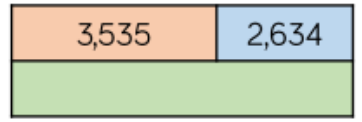
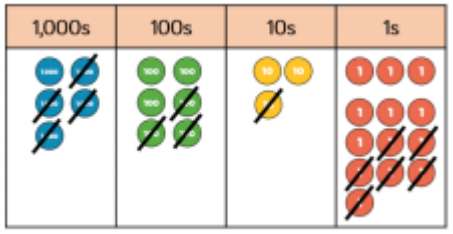
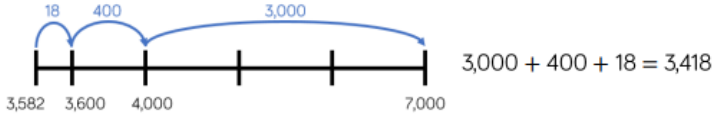
Year 1	Concrete	Pictorial	Abstract
Multiplication	Make arrays with objects such as cubes, in different orientations: xxx xx xxx xx xx Relate this to repeated addition. Use Numicon pieces of the same value to count up e.g. 2 pieces or 5 pieces, or 10 pieces.	Draw arrays on whiteboards in different orientations, for example using spots or crosses to represent objects. Relate this to repeated addition. Use pictorial representations of objects in 2s, 5s and 10s to work out problems e.g. 3 hands each with 5 fingers = 5+5+5 = 15,  or 5 bunches of 10 flowers = 10+10+10+10+10 = 50 	Count in multiples of 2, 5 and 10 and use this to solve problems. Write multiplications as repeated additions to solve a problem: Crayons come in packets of 5. I have 4 packets. How many crayons? 5+5+5+5 = 20 (solved by counting up in 5s)
Division	Share physical objects equally into groups, for example using cubes to represent cakes. If there are 12 cakes to be shared, count out 12 cubes. If there are 3 friends to share them, get 3 plates, and count out the cubes onto the plates in turn.	Draw objects or representations onto a whiteboard and draw around groups to share them out equally, seeing how many groups there are and how many are in each group. Care must be taken to ensure groups are equal.	Problem solving using sharing into groups. 

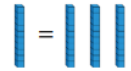
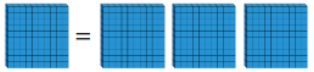
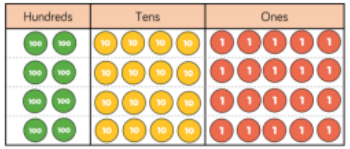
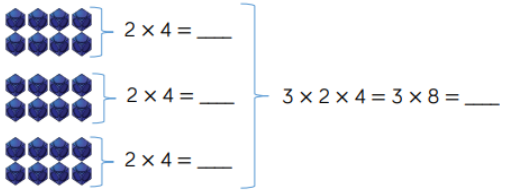
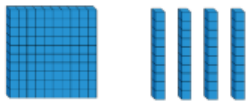
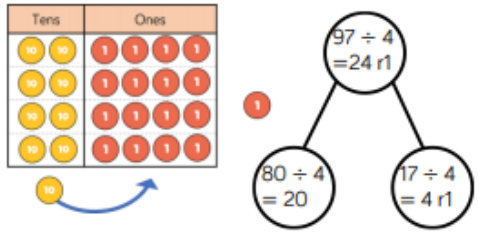
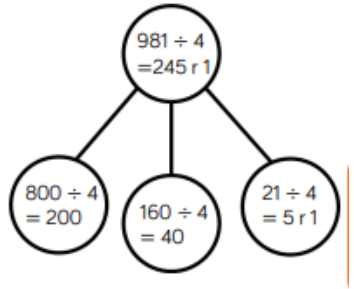
Year 2	Concrete	Pictorial	Abstract
Addition	Use tens frames and counters where one counter represents one 10 to support learning of 10s number bonds to 100. e.g. 30+70=100 Use Dienes to create two 2-digit numbers and combine tens and ones, initially without exchange and then with exchange (10 ones cubes exchanged for 1 ten stick) Use counters on place value grids to support adding a multiple of 10 to a 2-digit number or two 2-digit numbers. This can then be linked to children SEEING the format of column addition as it is visually very similar., but children in Year 2 do not record in this way. Dienes and a 100 square can be used together to work out bonds to 100 where they are not simple multiples of 10 (e.g. 54 and …?) Tens frames and counters to add three one-digit numbers (visual support to recognise bonds)	100 square to support addition of 10 (10 more) to any number Pictorial abacus (jotting) to represent a number in tens and ones so that a multiple of ten can be added. Number lines for adding a one-digit number to a two-digit number, including partitioning the one-digit number to cross the ten. E.g. 17+5 = 17+3+2. Exposure to a range of pictorial representations from which they must derive addition number sentences or solve problems. Draw two sets of sticks and bricks (lines and dots) to represent Dienes and to add up two 2-digit numbers.	Use a given bar model or part-whole to derive all related addition and subtraction facts and fact families for numbers to 20, writing horizontal number sentences with the total either at the beginning or the end of the number sentence. 16=10+6, 0r 10+6=16 Complete partial bar models and part-whole models and missing number sentences using numerals.  Use single digit bonds to derive tens bonds e,g. 4+6=10, so 40+60=100 Use estimation to check calculations. Look quickly for known number facts when adding three 1-digit numbers. Use part-whole models to partition 2-digit numbers into tens and ones and write horizontal number sentences to match.
Subtraction	Exchange to subtract a 1- or 2- digit number from a 2-digit number using Dienes e.g. make 24 with two tens and 4 ones. We need to subtract 8 so we must exchange one of our tens sticks for 10 ones.  Use counters on place value grids to support subtracting a multiple of 10 from a 2-digit number or two 2-digit numbers. This can then be linked to children SEEING the format of column subtraction as it is visually very similar., but children in Year 2 do not record in this way.	100 square to support subtraction of 10 (10 less) from any number. Exposure to a range of pictorial representations from which they must derive subtraction number sentences or solve problems. Draw sticks and bricks to represent a 2-digit number as Dienes and cross off/ rub out to take away another number (no exchange).	As above. For both addition and subtraction, exposure to missing number problems, including making comparisons of calculations. $45 - 17 > 14 + \_\_\_$ $26 + 15 < 60 - \_\_\_$

Year 2	Concrete	Pictorial	Abstract
Multiplication	Use physical arrays to explore commutativity of multiplication facts e.g. 5x2=2x5 Understand 'equal' groups and manipulate objects to create these in different ways. The Base 10 shows six equal groups with ten in each group. There are six tens.  How else can you represent these as equal groups?	Use pictorial representations to derive horizontal number sentences as repeated addition and also using the multiplication symbol.  ___ + ___ + ___ = 18 ___ × ___ = 18  4 lots of 3 = 1 × ___	Solve written problems including missing number problems and comparing multiplications and repeated additions. 3 × 5 ◯ 5 + 5 + 5 + 5 2 × 2 ◯ 2 + 2 10 × 2 ◯ 5 + 5 + 5 The total is 12, what could the addition and multiplication be? 3 × ___ = 6 ___ × 2 = 20 ___ = 8 × 2
Division	Use concrete materials to share amounts into equal groups, including sharing into 2, 5 and 10 groups to relate to dividing by 2,5,10 (linking this to knowledge of counting in 2s, 5s, 10s). Share into two groups to understand odd and even.	Use pictorial representations to derive horizontal number sentences using the division symbol. Apples can be sold in packs of 10 How many packs can be made below?  □ ÷ □ = □ Pencils come in packs of 20 We need to put 5 in each pot. How many pots will we need? There are ___ pencils altogether. There are ___ pencils in each pot. There are ___ pots.  	Solve written problems including missing number problems and word problems. Alex has 20 sweets and shares them between 5 friends.  Tommy has 20 sweets and shares them between 10 friends. Whose friends will receive the most sweets? How do you know?

Year 3	Concrete	Pictorial	Abstract
Addition	Use Dienes to add 1-digit numbers to 3-digit numbers USING EXCHANGE (i.e. crossing the 10). Use Dienes and place value counters with place value grids to support addition of 2-digit numbers to 3-digit numbers (initially multiples of 10, and initially without exchange, then with exchange). This method directly and visually supports the written column method. They will then move on to adding 3-digit numbers to 3-digit numbers (including multiples of 100) using place value counter and place value grids, initially without and then with exchange. 136+52 	Pictorial abacus (jotting) to represent a number in hundreds, tens and ones so that a multiple of 100 can be added. Use a number line to add 1-digit numbers to 3-digit numbers, partitioning the ones number to cross the 10 and so jumping in two steps. The partitioning of the ones number can be recorded alongside in a part-whole model to help children recall the two steps. Use bar models and part-whole models to support understanding of addition of 2- and 3- digit numbers to 3-digit numbers (varied fluency) Exposure to a range of pictorial representations from which they must derive addition calculations or solve problems. Draw representations of Dienes: squares (100s), sticks (10s) and bricks (1s) to solve additions.	Use mental arithmetic to add 1-digit numbers to 3-digit numbers, initially without exchange e.g. 532+45= ? Use the column method to add 2-digit numbers (initially multiples of 10) to 3-digit numbers, initially without exchange, and then with exchange: $\begin{array}{cccc} & 3 & 1 & 7 \\ + & & 4 & 6 \\ \hline & 3 & 6 & 3 \\ & & 1 & \end{array}$ Next use the same method to add 3-digit numbers to 3-digit numbers, again initially without and then with exchange.
Subtraction	Use Dienes to subtract 1-digit numbers from 3-digit numbers USING EXCHANGE (i.e. crossing the 10). Use Dienes and place value counters with place value grids to support subtraction of 2-digit numbers from 3-digit numbers (initially multiples of 10, and initially without exchange, then with exchange)., and then 3-digit numbers from 3-digit numbers (again initially without and then with exchange) 	Use a number line to subtract 1-digit numbers from 3-digit numbers, partitioning the ones number to cross the 10 and so jumping in two steps. The partitioning of the ones number can be recorded alongside in a part-whole model to help children recall the two steps.  Use a number line to support subtraction of 2-digit numbers from 3-digit numbers including jumping UP (finding the difference) Use bar models and part-whole models to support understanding of subtraction of 2- and 3- digit numbers from 3-digit numbers (varied fluency) and will be exposed to a range of pictorial representations from which they must derive subtraction calculations or solve problems.	Use mental arithmetic to subtract 1-digit numbers from 3-digit numbers, initially without exchange e.g. 356-5= ? Use the column method to subtract 2-digit numbers (initially multiples of 10) from 3-digit numbers, initially without exchange, and then with exchange: $\begin{array}{cccc} & {}^{3}\not{4} & {}^{1}3 & 4 \\ - & & 7 & 2 \\ \hline & 3 & 6 & 2 \end{array}$ Next use the same method to add 3-digit numbers to 3-digit numbers, again initially without and then with exchange.

Year 3	Concrete	Pictorial	Abstract
Multiplication	Use concrete materials to explore the 4 times table as doubling and doubling again, and the 8 times table as double this. Multiply a two-digit number by a one-digit number, using concrete materials to support this as repeated addition e.g. 21x3 = 21+21+21, which can be created with Dienes or place value counters. There are 21 coloured balls on a snooker table. How many coloured balls are there on 3 snooker tables? Use Base 10 to calculate: 21 × 4 and 33 × 3 	Use pictorial representations relating to the 3, 4 and 8 times table. Match the multiplication to the representation. 4 × 4 4 × 6 8 × 4 	Learn facts from the 3, 4 and 8 times tables by heart. Use what they know to find out new facts e.g. if 2x6=12, then 2x60=120, and complete fact families based on this. Use the formal column method to multiply a 2-digit number by a 1-digit number, initially without and then with exchange. 
Division	Divide 2-digit numbers by 1-digit numbers by partitioning into tens and ones using concrete materials and sharing into equal groups initially without exchange 84 ÷ 2  and then with ıg place value counters)	Use a part-whole model to partition the division e.g. 42÷3 is partitioned into 30÷3 and 12÷3 to make it more manageable. Annie uses a similar method to divide 42 by 3   Move on to division of a 2-digit number by a 1-digit number where there is a remainder, for example using a number line. Tommy uses repeated subtraction to solve 31 ÷ 4  31 ÷ 4 = 7 r 3	Understand division as the inverse of multiplication and use known multiplication facts for the 3, 4 and 8 times tables to derive division facts. Complete the bar models and the calculations.  24 ÷ 4 = ___ 80 ÷ 8 = ___ 64 ÷ 8 = ___ ___ × 8 = 24 8 = 72 ÷ ___ 8 × ___ = 40 ___ ÷ 8 = 7 Compare the statements using <, > or = 48 ÷ 4 ◯ 36 ÷ 3 52 ÷ 4 ◯ 42 ÷ 3 60 ÷ 3 ◯ 60 ÷ 4

Year 4	Concrete	Pictorial	Abstract
Addition	Add two 4-digit numbers initially without exchange, and then with exchange (one exchange only to begin with), using place value counters in a place value grid (with 1000s, 100s, 10s, 1s) 	Exposure to a range of pictorial representations from which they must derive addition calculations or solve problems. Draw representations of Dienes: cubes (1000s), squares (100s), sticks (10s) and bricks (1s) to solve additions.	Use the formal column addition method to add two 4-digit numbers, initially without and then with exchange. Exchange is experienced in different columns. See addition problems presented in different ways to sharpen their skills (varied fluency) including word problems, completing bar models and deciding whether statements are true or false and why.   Use inverse operations to check their calculations.
Subtraction	Subtract two 4-digit numbers initially without exchange, and then with exchange (one exchange only to begin with), using place value counters in a place value grid (with 1000s, 100s, 10s, 1s) 5643 - 4316 	Look at efficient methods for subtraction – consider using an EMPTY NUMBER LINE to count back, or crucially to count UP and when this might be a better strategy. 7000-3582  Draw representations of Dienes: cubes (1000s), squares (100s), sticks (10s) and bricks (1s) to solve subtractions. Exposure to a range of pictorial representations from which they must derive subtraction calculations or solve problems.	Use the formal column subtraction method to subtract two 4-digit numbers, initially without and then with exchange. Exchange is experienced in different columns.  See subtraction problems presented their skills (varied fluency) including word problems, completing bar models and deciding whether statements are true or false and why. Use inverse operations to check their calculations.

Year 4	Concrete	Pictorial	Abstract
Multiplication	Manipulate concrete materials to understand making something 10 or 100 times bigger i.e. multiplying by 10 or 100. 3 ×  = ___ tens = ___ 3 ×  = ___ hundreds = ___ Use place value counters to multiply a 3-digit number by a 1-digit number. A school has 4 house teams. There are 245 children in each house team. How many children are there altogether?  H T O 2 4 5 × 4	Use pictorial representation to understand multiplying by 1 or 0 Complete the sentences.  There are ___ plates. There is ___ banana on each plate. Altogether there are ___ bananas. ___ × ___ = ___ Use associative law to multiply 3 numbers.  2 × 4 = ___ 2 × 4 = ___ 2 × 4 = ___ 3 × 2 × 4 = 3 × 8 = ___	While in Wonderland, Alice drank a potion and everything shrank. All the items around her became ten times smaller! Are these measurements correct? Problem solving Item / Original measurement / After shrinking Height of a door / 220 cm / 2,200 cm Her height / 160 cm / 16 cm Length of a book / 340 mm / 43 mm Height of a mug / 220 mm / ? Can you fill in the missing measurement? Formal column method for multiplying a 3-digit number by a 1-digit number ir H T O 2 3 4 × 6 1 4 0 4 2 2
Division	Use concrete materials to understand making something 10 or 100 times smaller i.e. divide by 10 or 100 Use Base 10 to divide 140 by 10 Explain what you have done.  Use concrete materials (e.g. counters) to investigate dividing by 1 or 0. Use counters and hands to complete. • 4 counters **shared** between 4 hands ___ ÷ ___ = ___ • 4 counters **shared** between 1 hand ___ ÷ ___ = ___ • 9 counters **grouped** in 1s ___ ÷ ___ = ___ • 9 counters **grouped** in 9s ___ ÷ ___ = ___	Use pictorial representations to derive fact families for the 6, 7 and 9 times tables. Complete the fact family.  ___ × ___ = ___ ___ × ___ = ___ ___ ÷ ___ = ___ ___ ÷ ___ = ___ Part-whole models used alongside place value counters to divide a 2-digit number by a 1-digit number with remainders.  Tens Ones 97 ÷ 4 =24 r1 80 ÷ 4 = 20 17 ÷ 4 = 4 r1	Part-whole model to divide a 3-digit number by a 1-digit number (in this instance with a remainder)  981 ÷ 4 =245 r 1 800 ÷ 4 = 200 160 ÷ 4 = 40 21 ÷ 4 = 5 r 1